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Research interests

- Average values of $L$-functions
- Connection between $L$-functions and Random Matrix Theory
- Reciprocity formulae (and moments) for cotangent sums...
- ...and for twisted moments of Dirichlet $L$-functions
- Bounds for sums of Kloosterman sums
- Average root numbers for some families of Elliptic curves
- Distribution of $\sum_{n \leq Y} d(n)e(nx)$ as $x$ varies in $[0, 1]$ and $Y \to \infty$
Research interests

- **Average values of L-functions**
  - Asymptotic for $\sum_f L_f(\frac{1}{2} + \alpha)L_f(\frac{1}{2} + \beta)$ for some families of $L$-functions uniformity in the (large & vertical) shifts.
  - Asymptotic for moments of $L$-functions times arbitrary Dirichlet polynomial.
  - Asymptotic for
    $$\sum_{\chi_1,\ldots,\chi_k} |L(\frac{1}{2}, \chi_1) \cdots L(\frac{1}{2}, \chi_k)L(\frac{1}{2}, \chi_1 \cdots \chi_k)|^2$$

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- **Reciprocity formulae (and moments) for cotangent sums.**

\[
c_0(h/k) := - \sum_{m=1}^{k} \frac{m}{k} \cot(\pi mh/k)
\]

Reciprocity formula:
\[
c_0(h/k) + \frac{k}{h} c_0(k/h) - (\pi k)^{-1} = \psi(h/k),
\]
where $\psi$ is analytic.

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\[ M(a, q) := \frac{\sqrt{q}}{\varphi(q)} \sum_{\chi \mod q} |L(1/2, \chi)|^2 \chi(a) \]

Reciprocity formula:
\[ M(a, q) + M(q, a) + R(a, q) = \eta(h/k), \]
where $\eta$ is continuous (...) and $R$ is an explicit rational function.

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- **Bounds for sums of Kloosterman sums**

Bounds for sums of the form

$$\sum_{a,n,m} \alpha_m \beta_n \gamma_a \exp(2\pi i \frac{am}{n})$$

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**Average root numbers for some families of Elliptic curves**

- Study of some 1-parameter families of elliptic curves with non-zero average root-number
- Computation of the 1-level density for the $L$-functions of the elliptic curves in such families

- Distribution of $\sum_{n \leq Y} d(n)e(nx)$ as $x$ varies in $[0, 1]$ and $Y \to \infty$
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